



Essential Instructional Practices in Early Mathematics: Prekindergarten to Grade 3

This document was developed by the MAISA Early Mathematics Task Force, a subcommittee of the Michigan Association of Intermediate School Administrators (MAISA) General Education Leadership Network (GELN), which represents Michigan's 56 Intermediate School Districts.



INSTRUCTIONAL PRACTICES

What do you value as a teacher of young children?

Perhaps you value children's play — a child's unstructured, personally motivated engagement in an activity for joy. Or perhaps you value children's curiosity — a child's desire to understand how something works. Our values fundamentally shape our interactions with young children, as well as our expectations of their learning. Five core values, grounded in evidence from research, supported the development of the *Essential Instructional Practices in Early Mathematics: Prekindergarten to Grade 3*. These Essentials seek to promote high-quality, equitable teaching and learning for young children in the discipline of mathematics. By sharing these core values, we pull back the curtain so all can understand the backdrop against which the Essentials take center stage.

Our Values

1. **We value** children seeing themselves as mathematics knowers, doers, and contributors to the field of mathematics.
2. **We value** children's differences with respect to age, race, ethnicity, gender, socio-economic status, and geographic context (e.g., urban, rural, suburban).
3. **We value** mathematics as a broad, creative, and collaborative discipline for sense making and problem solving.
4. **We value** learning mathematics in a variety of ways — both socially and cognitively.
5. **We value** mathematics teaching that focuses on connection, care, and authentic relationships.

We encourage you to read the instructional practices through the lens of these values. **See p.7 for detail about each core value.**

Purpose

This document is a tool intended to support educators across Michigan as we work to enhance the ways in which children learn to use, understand, and do mathematics using a strengths-based approach. The long-term goal is that these *Essential Instructional Practices in Early Mathematics: Prekindergarten to Grade 3* will prompt shifts in systems, learning, teaching, and assessment so that each and every child develops strong early mathematical understanding, skills, and dispositions. Toward this end, this document includes a small set of research-supported instructional practices that are consistent with and built upon recommendations from the [National Council of Teachers of Mathematics \(NCTM\)](#), [National Association for the Education of Young Children \(NAEYC\)](#), and [National Research Council \(NRC\)](#). Just as Carpenter et al. (2017) assert, it is time that a reframing of mathematics teaching and learning takes place that “goes beyond debates about telling versus not telling, discovery versus direct instruction, play-based versus structured” (p. 5). We hope this document will serve as a guidepost to shift educators away from such debates and toward a focus on working together and with families to enhance the ways in which we support, recognize, and use children’s mathematical thinking and the varied ways in which children express their thoughts.

The instructional practices outlined in this document should be used as the focus of continuous improvement efforts, inclusive of professional learning and systemic supports, designed to enhance and sustain productive, joyful learning environments for children and educators. Research suggests that these eight practices have significant potential to positively affect children’s learning of mathematics and the development of their mathematical identities. We believe that use of these practices will set our state on a path to make a measurable positive difference in mathematics achievement in Michigan, as well as contribute to the development of high-quality STEM experiences, as recommended by the MiSTEM Advisory Council, April 2018 Report.

The eight Essential Instructional Practices are strategic, supported by evidence from research and practice, and interconnected in many ways. Taken together, these practices are stronger than any single practice. Still, as research evolves, it may become necessary to add to or alter the instructional practices recommended herein. Given the crucial nature of these practices and the relationships among them, they should occur regularly during instructional time, and should be focused on during



mathematics instruction periods and throughout the day during more “organic” mathematical opportunities. Mindfulness of naturally emerging opportunities as children interact is an essential piece in helping children see mathematics as part of their world.

We have organized the document as follows:

- Essential Instructional Practices 1 through 3 focus on the overall design of the learning environment and general ways of interacting within the learning environment.
- Essential Instructional Practice 4 focuses on the formative assessment process, a practice that should be intentionally and continually embedded throughout learning.
- Essential Instructional Practices 5 through 7 focus on the types of mathematical tasks, as well as routines for using these tasks to support deep and meaningful learning of mathematics.
- Essential Instructional Practice 8 highlights productive and purposeful ways of engaging children’s families/ caregivers as partners in the learning process.

The practices listed can be used within a variety of overall approaches to mathematics instruction and many different structures of the day — the document does not specify one particular instructional program or textbook series. Instead, this document promotes an instructional stance focused on a relentless curiosity to make sense of what children know and then uses children’s thinking and their ways of making sense as building blocks for future learning. In doing so, educators work to meet children where they are by

providing appropriate contexts and learning activities. Understanding this, it is critical to recognize that the inherent design of a mathematical task (i.e., how it appears in an instructional resource) has implications for children’s learning. In particular, tasks that are designed at higher levels of cognitive demand are more likely to be used with children in ways supportive of learning mathematics, rather than just encouraging students to produce answers (Stein and Lane, 1996). As such, selection of instructional resources is a critical piece to building a system that supports and enables teachers as they work with children in ways consistent with the practices outlined in this document.

These practices do not exist in a vacuum. Educators’ effective use of these practices will be significantly enhanced by a deep understanding of:

- early mathematics content;
- the ways in which children make sense of mathematics; and
- systemic inequities commonly reproduced in schools and classrooms.

“Mathematical ideas that are suitable for preschool and the early grades reveal a surprising intricacy and complexity when they are examined in depth” (NRC, 2009, p. 21). Many educators have not been given adequate opportunities to learn about the intricacies and complexities of early mathematics. With this in mind, we recommend that as educators engage in professional learning focused on these instructional practices, they also engage with mathematics — particularly early number and geometry concepts, two fundamental areas of early mathematics (NRC, 2009; IES, 2013). In addition, opportunities to learn to recognize and remediate systemic inequities have the potential to influence educators’ efforts to support children as they learn mathematics.

Our goal is that these practices become habits of mind for educators. As learners (both children and adults) engage with these practices, learners may find that their perceptions of themselves mathematically — their mathematical identities — vary as they persevere to become increasingly competent and confident in their understanding of and ability to use mathematics. This document and the additional tools and resources to grow out of this initial work will support each of us to deepen and extend the ways in which we see mathematics and see ourselves within the context of mathematics.

In closing, it is important to read this document in relation to the [Michigan Early Childhood Standards of Quality for Prekindergarten](#) and the [Michigan K-12 Standards for Mathematics](#) (Kindergarten through Grade 3).

1. Design learning environments to encourage mathematical play and tinkering.

- Provide activities and tasks that encourage children to experiment with and explore mathematical ideas. These tasks should encourage:
 - choice;
 - creativity; and
 - social engagement.
- Provide access to a variety of materials (e.g., blocks, art supplies, manipulatives, measuring tools, games, puzzles, etc.) to prompt exploration of mathematical ideas.
- Flexibly use space during mathematical activities and tasks.
- Look for and highlight mathematical ideas in children’s play to help them describe and make sense of mathematics as part of their cultural and social worlds.
- Incorporate objects and pictures to promote spatial reasoning, measurement, and quantity, and to intentionally use mathematically accurate language when describing these things.
- Incorporate (e.g., display) diverse representations of people (e.g., race, gender, culture, age) doing a variety of mathematical work.
- Encourage children’s movements as they work to make sense of mathematical ideas (e.g., before and after, more or less).
- Allow children choices in how they position their bodies, such as sitting, standing, or lying down, as they engage with mathematical ideas.
- Explore the relationship of mathematics with other domains by intentionally infusing mathematics in other content (e.g., social studies, music, technology, dance, science, literacy, dramatic play, block play, art, etc.)

2. Establish and monitor norms (i.e., ways of being in a learning environment) with children to develop a classroom culture and climate that **promotes positive, robust mathematics identities.**

- Provide children with opportunities to see themselves as mathematicians (i.e., people who can use, do, and understand mathematics).
- Focus on growth, support productive struggle, and encourage children’s internal desire to learn, as opposed to external measures of achievement or rewards.
- Develop an awareness of how mathematics is expressed in the children’s communities, as well as in different communities.
- Ask and explore mathematical questions relevant to their world (e.g., “Which professions are represented in the books in our learning environment? How often is each profession represented?”).
- Use children’s cultural and personal background experiences (e.g., ethnic, racial, religious, extra-curricular, etc.) while exploring mathematical ideas. For example:
 - books with relevance to children’s lives and culture(s); and
 - objects and pictures from a child’s real-life experience, such as items from nature, common things seen in the child’s community, etc.
- Explore how mathematicians from diverse communities have played a role in the development of mathematical knowledge. For example:
 - incorporate children’s books to highlight stories of people who have used mathematics.
- Solicit differences in mathematical thinking and personal experiences and strategically leverage these as resources in learning (e.g., when working in groups, preparing the learning environment, offering examples or explanations).
- Value partial and potentially incorrect understanding of mathematical concepts and procedures as tools for highlighting productive thinking, including opportunities to scaffold learning.
- Avoid instructional experiences that potentially lead to anxiety (e.g., timed tests, around-the-world, etc.).
- Incorporate needs-based instruction, inclusive of flexible group structures; in doing so, avoid ability grouping that is long term and static in nature, as well as hierarchical language, as these practices can perpetuate low expectations and undermine future learning opportunities.

3. Ensure **equitable participation** of children in mathematics.

- Monitor children’s talk and intervene to create space for each child to express ideas by helping some children pause when needed, while encouraging others to contribute, particularly in relation to historical patterns of marginalization.
- Use strategies to promote wide participation (e.g., turn-and-talks, fingerplays, clapping or stomping rhythmic patterns, hand signals, soliciting multiple answers and/or strategies).
- Use a variety of participation structures (e.g., small groups, independent work, whole group) and representational contexts (e.g., graphs and diagrams, various ways of articulating numbers — verbal, quantity, symbolic, empty number lines, etc.) to support language development and create diverse opportunities for building and showing competence.
- Use sentence stems to support children’s engagement in mathematical discussions (e.g., “I agree/disagree because...”; “That solution is like/unlike mine because...”; “My answer is different because...”; “We could try...”; “Why did you...?”; “What if...?”).
- Model listening and communication using mathematical language (e.g., specialized vocabulary and terms) and support children in moving among more and less sophisticated mathematical language as they are developing ideas.
- Recognize and value children’s primary languages, developing proficiency in English, integrated use of multiple languages (i.e., translanguaging), and mode of communication (e.g., aided and/or unaided augmentative and alternative communication [AAC]) including the use of devices, gestures, images, and/or objects while learning.
- Allow and support children in making choices about mathematical tools and numbers as they work to solve problems. For example, children may use:
 - dot dice as opposed to those labeled with written symbols; and
 - numbers within one hundred rather than numbers within twenty.
- Distribute materials to support equitable engagement in group work using strategies such as:
 - providing enough materials so all children can engage with the task;
 - ensuring all children have access to task directions; and
 - providing limited materials (e.g., only giving some information to each child in a group) so that group members need to rely on one another while working.
- For group work, choose group-worthy tasks that require multiple mathematical abilities and the full engagement of multiple children.
- Structure group work to enable children with different levels of understanding or at different developmental levels to work together, with teacher support as needed.

4. Engage in formative assessment as a process — in an ongoing and planned-for manner, continuously assess children’s mathematical thinking through observation and discussion to inform the next learning and teaching steps.

- Use standards and previous evidence of children’s understanding to select daily mathematics activities that offer opportunities to notice children’s informal and formal ways of speaking about, representing, and doing mathematics.
- Look for and recognize different ways in which individual children demonstrate mathematical competence, including use of gestures, talk, representations, etc.
- Pose purposeful questions in order to elicit evidence of children’s thinking in various participation structures (e.g., individuals, pairs, and small and large groups).
- Support children in reflecting upon and communicating their own learning within and outside of the classroom.
- Use children’s current levels of understanding to provide timely, productive feedback and advance learning.
- Work from children’s identified strengths as the starting points for new mathematical learning and selection of appropriate learning tasks.
- Make instructional decisions guided by children’s current levels of understanding and by evidence-based learning trajectories/progressions (LT/P) to help children develop more sophisticated solution strategies over time.
- Ensure that classroom-level assessment practices disrupt historical patterns of marginalization with respect to race, gender, ability, socio-economic status, and language.

5. Intentionally select and implement cognitively demanding mathematical tasks from instructional resources.

Cognitively demanding tasks are designed (as they appear in a resource or as planned by teacher[s]) to:

- ∴ hold high expectations for every child, leading to deep learning, by focusing on big ideas in mathematics (e.g., composing and decomposing numbers, reasoning about properties of various shapes, etc.);
- ∴ engage children in mathematical reasoning and sense making, both individually and collaboratively;
- ∴ allow multiple entry points, suggesting children work in a variety of mathematically productive ways; and
- ∴ typically relate and be relevant to children’s lived experiences.

- Establish goals focused on helping children develop mathematical understandings as opposed to simply answer getting.
- Launch the task in a manner that ensures access to every child by:
 - clarifying task expectations;
 - encouraging children to draw on their own lived experiences, as well as be resources for one another; and
 - building children’s knowledge of the context when a situation is unfamiliar to their lived experiences.
- Encourage and support children’s perseverance in problem solving in language-rich environments. For example:
 - ask open-ended questions;
 - ask focused questions to guide children through their problem-solving processes; and
 - prompt peer-to-peer mathematical talk (e.g., “Say in your own words what your friend just shared.”).
- Make children’s thinking visible around strategies and ideas, positioning each child as a valuable contributor. For example:
 - strategically select and sequence children’s thinking and representations for use with the whole class;
 - invite children to share artifacts from play and/or outside of school; and
 - recognize various ways in which strategies and ideas may be shared (e.g., through movement, talk, images, symbols, and children’s stories).
- Support children in making connections among strategies and representations. For example:
 - link work on an empty number line to a more formally notated computation strategy;
 - represent a written number story using objects or pictures;
 - create an array as a model for a multiplication number sentence; and
 - write a story context that fits the array.

6. Engage children regularly in brief (5-10 minute) interactive number-sense routines focused on developing mental strategies for seeing quantity and working flexibly with numbers.

Brief interactive number-sense routines typically include the following steps:

- ✦ the teacher poses a visual, verbal, and/or written mathematical prompt (e.g., “How many dots did you see?”);
- ✦ children think individually;
- ✦ supported by the teacher, children share thinking, including non-fully formed ideas;
- ✦ the teacher notates and/or verbalizes the children’s strategies; and
- ✦ the learning community discusses the thoughts that emerged and works to draw conclusions.

- Enable children to engage in sense making mentally by selecting accessible prompts.
- Encourage children to develop their own strategies to solve these problems in ways that make sense to them without telling children a specific procedure to use.
- Elicit children’s thinking by asking them to share their strategies, discuss the strategies of others, and make connections among multiple strategies.
- Support children in paraphrasing one another’s thinking to foster communication and language development.
- Notate children’s strategies to make ideas accessible to classmates and to encourage movement toward increasingly flexible and sophisticated mental strategies.
- Emphasize sense making and de-emphasize speed.

7. Engage children regularly in making sense of and solving story-based problems, both those that are planned for and those that come up organically throughout the day.

- Provide children access to and support them in making sense of a variety of real-world problems with varied structures (e.g., “We have two crayons at our table. How many more do we need for all five of us to have a crayon?”; “You have two crayons and I have three crayons — how many do we have altogether?”).
- Select problems grounded in accessible and relevant contexts to children by:
 - using child-generated stories to create mathematical problems;
 - enabling children to make mathematical connections using examples from their community or home environment;
 - empowering children to connect mathematical concepts, such as more than, less than, same as, equal to, and fair shares with issues of fairness in their everyday lives; and
 - building children’s knowledge of context when a situation is unfamiliar to their lived experiences, to help gain access to the mathematics.
- Observe and identify children’s solution strategies (i.e., thinking and processes children use while engaging in mathematical work, not just how they represent their thinking) and use these observations to inform the selection of future problems. Strategies might include the use of:
 - direct modeling;
 - counting;
 - derived facts;
 - children’s invented strategies and/or algorithms; and
 - standard algorithms.

8. Learn from and support families in promoting children’s mathematical thinking.

- Enact practices to make all children and families feel welcome in mathematics classrooms (e.g., respond sensitively to questions and concerns, recognize the demands homework places on family time and relationships, write and talk about mathematics in accessible ways).
- Learn about family activities, hobbies, and cultural practices that may relate to mathematics and incorporate these activities into the classroom community daily or during special events.
- Engage families and children in positive mathematics experiences (e.g., family nights at school that include mathematics games or activities, field trips that explore mathematical ideas, etc.).
- Be sensitive to families’ language practices in all communications and suggestions for out-of-school activities.
- Provide ideas for out-of-school mathematical experiences, such as providing games or other resources that can be used in fun and engaging ways.
- Engage families to gather feedback on school mathematics experiences to guide future interactions and to promote positive experiences for each child.
- Communicate frequently with families, using an asset-based approach, to celebrate successes and identify strategies to achieve future goals.

Essential Instructional Practices for Early Mathematics

Core Values



Value #1: Developing positive, robust mathematics identities — how children see themselves

Children’s mathematics learning requires their own personal sense making as they engage in, construct, and acquire understandings, skills, and competencies in various mathematical domains, such as number and shape. Also critically important to a child’s mathematics learning is their identity development as a person who knows and does mathematics. Too often children engage in various mathematical activities but fail to see themselves as a mathematics knower and doer. So many children and adults readily state, “I’m not a math person.” But the reality is that we are all math people because we all engage in mathematizing every day (counting, estimating, seeking patterns, and problem solving).

Identity development is fundamentally about creating positive and affirming relationships with mathematics and seeing oneself as competent. Developing a relationship with mathematics is in many ways like developing a friendship. Just as personal relationships unfold differently

among individual children, each child develops their relationship with mathematics differently. In the end, a positive relationship is critical to engagement in the field of mathematics — whether during an activity in class today or sustaining interest and perseverance in mathematics for years to come. We know many children develop fragile relationships with mathematics, relationships which may begin to fracture when a child receives messages, often unintended, that make the child feel unable to do, unwelcome to participate in, and/or unsupported to engage with mathematics. Avoiding challenging mathematical activities and tasks, however, does nothing to grow and strengthen that relationship. Rather, developing a positive, robust relationship — a productive disposition toward mathematics — involves consistently providing meaningful challenges and adequate support so children can grow and strengthen a positive, robust mathematics identity.

Value #2: Embracing social differences as assets to learning and doing mathematics

What images come to mind when you think of a scientist or a mathematician? For many, the images we have consist of common characteristics that paint a limiting picture of who participates in science and mathematics. Since 1957, in the draw-a-scientist-test (DAST), children irrespective of race, gender, and class have typically drawn images of a man with lab coat, glasses, and facial hair holding different scientific tools. Perhaps not surprisingly, the results are quite similar when asked to draw a mathematician. Decade after decade, these studies reveal that even young children hold stereotypical images of scientists and mathematicians. Seeing oneself as a mathematician can be daunting when one doesn't see "people like me" among the predominant images around them at school and in society.

The example above is centered on visible qualities, but social differences are not only perceived visually; changing what we picture with respect to who participates in science and mathematics is only part of embracing social differences as assets to learning and doing mathematics. In addition to such visible qualities, we must also recognize and embrace other social differences, such as the varied ways in which children communicate and interact as they learn mathematics.

Children bring a range of their own socio-cultural identifiers, or social markers, with them into their mathematics learning. Each of these markers holds different personal and societal histories. In learning and practicing mathematics, many markers of race, gender, and class have long been used formally and informally to enable access to only a few, while restricting access to others. Children's competencies in learning mathematics are not and should not be determined by their social markers. Still, these social markers can influence each person's perception of what it means to know and do mathematics, as well as who can and should do mathematics. So that all children can see themselves as learners and doers of mathematics, we believe that embracing the differences that all children naturally carry with them can only bring assets and resources to enrich learning and the relationships that students grow with each other, with their teachers, and with mathematics. Both recognizing and valuing these social differences as we work to broaden the images of competent mathematics knowers and doers are critical to ensure that every child sees themselves among images both appealing and empowering.

Value #3: Expanding the meaning of mathematics — a broad, creative, collaborative discipline based on sense making and problem solving

How do you remember your experiences in mathematics? Too often, mathematics has been experienced at school as a rigid discipline dependent on speed and computational correctness. Many people's lasting memories involve timed tests and frustration that resulted. Others' memories of school mathematics include a sense of safety and comfort that there was always a correct answer if a set of procedures was just applied systematically — although not necessarily with an understanding of why or how they worked. Speed and accuracy have a long history of being the defining characteristics of school mathematics. Regrettably, this has unnecessarily created a small number of children and adults who are positioned as smart/winner and a large number of children and adults who are positioned as not smart/losers in the "math game." Witness the widespread perception that only a few people are "math people" and most people simply are not — and this situation is often seen as not only fine, but expected.

While mathematics operates within the bounds of logic toward well-reasoned results, mathematics as a discipline is a dynamic field of study that invites broad, creative, argumentative, and collaborative thinking. Mathematics can help children make sense of their everyday world through numbers, patterns, shapes, and logical reasoning. Young children arrive at school already having developed some ideas on their own, particularly in regard to numbers and shapes. These everyday understandings of mathematics can serve as the foundation for children to continue developing their mathematical intuitions, and to gradually build toward abstract concepts that may or may not directly connect to the physical world. Mathematics encourages children to ask "what if?" — to conjecture and then verify mathematical ideas. Learners' first and subsequent experiences with mathematics can be premised on inquiry, discovery, and connection making that are challenged and verified within a classroom community through discussion and play. We therefore value a view of mathematics that is broad and creative, and focuses on meaning making in a learning community.

Value #4: Children learn mathematics in different ways toward the same ends

Imagine a group of children at a playground. Children are playing and engaging with each other in a variety of ways. Some use the different play structures as designed, gliding down the slide, while others are intent on seeing what else is possible, climbing up the slide instead. Some children require support from their friends or an adult to climb up ladders or swing on the swings, whereas others navigate challenges on their own. A few children are watching the others until they're ready to join in, and still another group of children is exploring the play structure, tinkering in creative ways and following wherever their imaginations take them.

Learning mathematics similarly reflects this diversity in both cognitive and social processes. Children express their abilities, intelligences, needs, and interests through

a variety of forms and modes of interaction. Mathematics learning can be embraced as a different kind of playground that also invites and encourages a wide variety of abilities and styles of interaction. Furthermore, just as no child's style or preference for play is better than another's, modes for mathematics learning are not placed along some quality hierarchy. In fact, as a child works to make sense of a mathematical idea or set of ideas, this child may move from one mode to another given the particulars of the context. As with social markers, these differences in participation and engagement preferences are valued resources that create a dynamic, mathematically rich, playful, and joyful experience for all with abundant action, sound, and feeling.

Value #5: Teaching practices that value and honor children's humanity

Building relationships with children is critical to being able to use teaching practices that value and honor children as unique persons in their own right, not as empty vessels to be filled. Relationship-building teachers express curiosity in their children and take time to find out: Who is this young person? Why might they be thinking this way? How might we work together in the next steps for growth? Regrettably, all too often, the aims and objectives of even the best teaching practices become narrowed — intentionally or not — to focus on covering mathematical content and managing classroom behavior. When this occurs, it tends to keep children at

arms length from mathematics and from each other, often minimizing the critical interpersonal dimensions of meaning making and compromising the learning that should last a lifetime. This narrowed focus may also prompt teachers to be more corrective than curious when considering the next instructional steps and when viewing children's work. We value and privilege teaching practices that place the physical and socio-emotional aspects of children's development in the foreground, focusing on the human connection, on the care of others along with self, and on the building of authentic positive relationships — while simultaneously developing mathematics content knowledge.

Concluding thoughts on teaching practices...

Teaching practices are more than tools for which we intentionally plan or may improvise their use in the moment; they have the power to communicate what it might look and sound like to know and do mathematics. Teaching practices have histories — what these practices have meant in the past for children and communities, as well as futures — what these practices hope to accomplish in the coming weeks, months, and years. Another way to think about teaching practices is to consider the cumulative effect of a practice over time and to professionally and personally reflect: How has the practice been used in the past? What sort of relationships did this practice create between a child and mathematics, or between children? If this practice is continued, what sort of relationships might this practice create between a child and mathematics or between children in the years to come? Given the power that teaching practices wield, care must be taken to privilege

those practices that drive not just content-learning, but that also grow positive identities, equitable access, and quality relationships.

Clearly, teaching young children mathematics is complex. “Teaching is what teachers do, say, and think with learners, concerning content, in particular organizations and other environments, in time.” (Cohen, Raudenbush, & Ball, 2003, p. 124). Doing it effectively requires navigating the interactions described above while attending to the physical, socio-emotional, and intellectual needs and interests for not only one child, but for several children simultaneously. In short, we value teaching practices that engage children as partners in learning meaningful and useful mathematics content, inclusive of mathematical practices, and that honor children as individual and unique persons.

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Process for Development and Review

This document was developed by the MAISA Mathematics Task Force, a subcommittee of the Michigan Association of Intermediate School Administrators (MAISA) General Education Leadership Network (GELN), which represents Michigan's 56 Intermediate School Districts. The Task Force included representatives from the following organizations, although their participation does not necessarily indicate endorsement by the organization they represent:

Allegan Area Educational Service Agency	Michigan Council of Teachers of Mathematics
Alt+Shift	Michigan Department of Education
Bay-Arenac Intermediate School District	Michigan Mathematics and Science Leaders Network
Calhoun Intermediate School District	Michigan State University
Eaton Regional Educational Service Agency	Macomb Intermediate School District
Grand Valley State University	Mecosta Osceola Intermediate School District
Gratiot-Isabella Regional Educational Service District	MiSTEM Network
Ingham Intermediate School District	Monroe County Intermediate School District
Iosco Regional Educational Service Agency	Montcalm Area Intermediate School District
Kalamazoo Regional Educational Service Agency	Muskegon Area Intermediate School District
Kent Intermediate School District	Oakland Schools
MAISA Early Childhood Administrators Network	Saginaw Intermediate School District
MAISA Mathematics Leadership Team	Shiawassee Regional Educational Service District
Michigan Assessment Consortium	University of Michigan
Michigan Association of Mathematics Teacher Educators	Washtenaw Intermediate School District
Michigan Association of Superintendents & Administrators	Wayne Regional Educational Service Agency

Feedback on drafts of the document was elicited from other stakeholders, resulting in a number of revisions to the document.

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